

S. 66/6

(1)

$$a) f(x) = x^4 - 6x^2 + 1; f'(x) = 4x^3 - 12x$$

$$\text{h.B. } 0 = x(4x^2 - 12)$$

$$x_1 = 0$$

$$x_{2,3} = \pm\sqrt{3}$$

<u>h.B.</u>	$J_1(-2 -\sqrt{3})$	$f'(x) = x(4x^2 - 12) < 0$	} P_{T_1}
	$J_2(-\sqrt{3} -1)$	$f'(x) = x(4x^2 - 12) > 0$	
	$J_3(-1 0)$	$(-) \cdot (-) > 0$	} P_H
	$J_4(0 1)$	$(+) \cdot (-) < 0$	
	$J_5(1 \sqrt{3})$	$(+) \cdot (-) < 0$	} P_{T_2}
	$J_6(\sqrt{3} 2)$	$(+) \cdot (+) > 0$	

$$\wedge P_{T_1}(-\sqrt{3} | -8); P_H(0 | 1); P_{T_2}(\sqrt{3} | -8)$$

$$b) f(x) = x^5 - 5x^4 - 2; f'(x) = 5x^4 - 20x^3$$

$$\text{h.B. } \wedge x_1 = 0; x_2 = 4$$

<u>h.B.</u>	$J_1(-1 0)$	$f'(x) = 5x^3(x-4)$	> 0	} P_H
	$J_2(0 4)$	$(+) \cdot (-) < 0$	< 0	
	$J_3(4 5)$	$(+) \cdot (+) > 0$	> 0	} P_T

$$\wedge P_H(0 | -2); P_T(4 | -258)$$

$$c) f(x) = x^3 - 3x^2 + 1; f'(x) = 3x^2 - 6x$$

$$\text{h.B. } \wedge x_1 = 0; x_2 = 2$$

<u>h.B.</u>	$J_1(-1 0)$	$f'(x) = 3x(x-2)$	> 0	} P_H
	$J_2(0 2)$	$(+) \cdot (-) < 0$	< 0	
	$J_3(2 3)$	$(+) \cdot (+) > 0$	> 0	} P_T

$$\wedge P_H(0 | 1); P_T(2 | -3)$$

$$d) f(x) = x^4 + 4x + 3; f'(x) = 4x^3 + 4$$

$$\underline{uB} \quad x_1 = -1$$

$$\underline{hB} \quad J_1(-2|-1): f'(x) = 4(x^3+1)$$

$$J_2(-1|0)$$

$$\begin{array}{ccc} (+) & (-) & < 0 \\ (+) & (+) & > 0 \end{array} \left. \vphantom{\begin{array}{ccc} (+) & (-) & < 0 \\ (+) & (+) & > 0 \end{array}} \right\} P_T$$

$$\wedge \quad P_T(-1|0)$$

$$e) f(x) = 2x^3 - 5x^2 + 12x - 4; f'(x) = 6x^2 - 10x + 12$$

$$\underline{uB} \quad x_1 = 1, x_2 = 2$$

$$\underline{hB} \quad J_1(0|1): f'(x) = 6(x-1)(x-2)$$

$$J_2(1|2)$$

$$J_3(2|3)$$

$$\begin{array}{ccc} (+) & (-) & (-) & > 0 \\ (+) & (+) & (-) & < 0 \\ (+) & (+) & (+) & > 0 \end{array} \left. \vphantom{\begin{array}{ccc} (+) & (-) & (-) & > 0 \\ (+) & (+) & (-) & < 0 \\ (+) & (+) & (+) & > 0 \end{array}} \right\} \begin{array}{l} P_H \\ P_T \\ P_T \end{array}$$

$$\wedge \quad P_H(1|1); P_T(2|0)$$

$$f) f(x) = (x^2 - 1)^2; f'(x) = 2(x^2 - 1)2x = 4x(x^2 - 1)$$

$$\underline{uB} \quad x_1 = 0, x_2 = 1, x_3 = -1$$

$$\underline{hB} \quad J_1(-2|-1): f'(x) = 4x(x^2 - 1)$$

$$J_2(-1|0)$$

$$J_3(0|+1)$$

$$J_4(1|2)$$

$$\begin{array}{ccc} (-) & (+) & < 0 \\ (-) & (-) & > 0 \\ (+) & (-) & < 0 \\ (+) & (+) & > 0 \end{array} \left. \vphantom{\begin{array}{ccc} (-) & (+) & < 0 \\ (-) & (-) & > 0 \\ (+) & (-) & < 0 \\ (+) & (+) & > 0 \end{array}} \right\} \begin{array}{l} P_{T_1} \\ P_H \\ P_{T_1} \end{array}$$

$$\wedge \quad P_{T_1}(-1|0); P_H(0|1); P_{T_2}(1|0)$$

7) $f(x)$	not. B.	krit. Bed.
a) $= x^2 - 5x + 5$	$0 = 2x - 5$ $x_E = \frac{5}{2}$	$f''(x) = 2$ $f''(\frac{5}{2}) = 2 > 0$ $\rightarrow P_T(\frac{5}{2} -\frac{5}{4})$
b) $= 2x - 3x^2$	$0 = 2 - 6x$ $x_E = \frac{1}{3}$	$f''(x) = -6$ $f''(\frac{1}{3}) = -6 < 0$ $\rightarrow P_H(\frac{1}{3} \frac{1}{3})$
c) $= x^3 - 6x$	$0 = 3x^2 - 6$ $x_{E_{1,2}} = \pm\sqrt{2}$	$f''(x) = 6x$ $f''(\sqrt{2}) = 6\sqrt{2} > 0$ $f''(-\sqrt{2}) = -6\sqrt{2} < 0$ $P_T(\sqrt{2} -4\sqrt{2})$ $P_H(-\sqrt{2} 4\sqrt{2})$
d) $= x^4 - 4x^2 + 3$	$0 = 4x^3 - 8x$ $x_{E_1} = 0$ $x_{E_{2,3}} = \pm\sqrt{2}$	$f''(x) = 12x^2 - 8$ $f''(0) = -8 < 0$ $f''(\sqrt{2}) = 16 > 0$ $f''(-\sqrt{2}) = 16 > 0$ $P_H(0 3), P_T(\sqrt{2} -1)$ $P_T(-\sqrt{2} -1)$
e) $= \frac{4}{5}x^5 - \frac{10}{3}x^3 + \frac{9}{4}x$	$0 = 4x^4 - 10x^2 + \frac{9}{4}$ $x_{E_{1,2}} = \pm\frac{3}{2}$ $x_{3,4} = \pm\frac{1}{2}$	$f''(x) = 16x^3 - 20x$ $f''(\pm\frac{3}{2}) = \pm 24 \geq 0$ $f''(\pm\frac{1}{2}) = \mp 8 \leq 0$ $P_H(-\frac{3}{2} \frac{9}{5}), P_T(\frac{3}{2} -\frac{9}{5})$ $P_H(\frac{1}{2} \frac{11}{15}), P_T(-\frac{1}{2} -\frac{11}{15})$

$$f) P_H(\sqrt{3} | -48\sqrt{3}), P_T(-\sqrt{3} | 48\sqrt{3})$$

$$g) a) f(x) = x^3 - ax$$

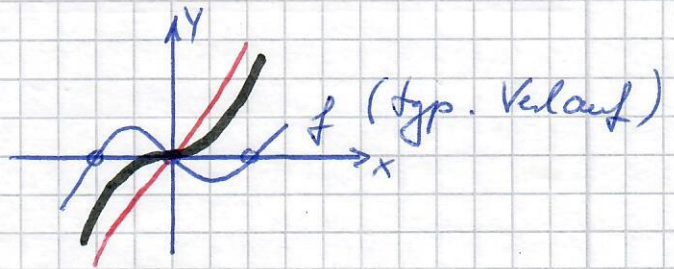
$$f'(x) = 3x^2 - a$$

$$\text{u.B. } 0 = 3x^2 - a$$

$$a = 3x^2$$

$$x^2 = \frac{a}{3}$$

$$\underline{\underline{x_{E_{1,2}} = \pm \sqrt{\frac{1}{3}a}}}$$



falls $a < 0$ \rightarrow keine wagg. T.

falls $a = 0$ $\rightarrow x_E = 0$

\rightarrow eine wagg. Tang.

falls $a > 0$ $\rightarrow x_{E_{1,2}}$

\rightarrow zwei wagg. T.

$$b) f(x) = x^4 + ax^2$$

$$f'(x) = 4x^3 + 2ax$$

$$\text{u.B. } 0 = 4x^3 + 2ax$$

$$0 = x(4x^2 + 2a)$$

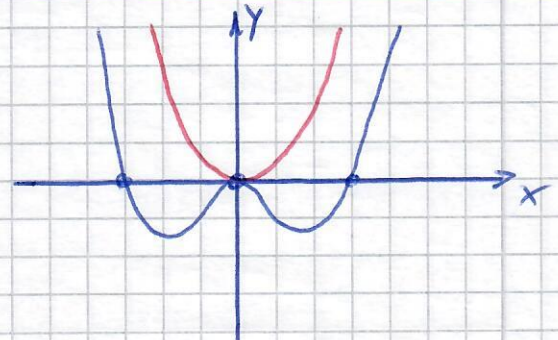
$$\underline{\underline{x_{E_1} = 0}}$$

$$0 = 4x^2 + 2a$$

$$4x^2 = -2a$$

$$x^2 = -\frac{1}{2}a$$

$$\underline{\underline{x_{1,2} = \pm \sqrt{-\frac{1}{2}a}}}$$



falls $a = 0$

\rightarrow eine wagg. T.

falls $a < 0$

\rightarrow drei wagg. T.

falls $a > 0$

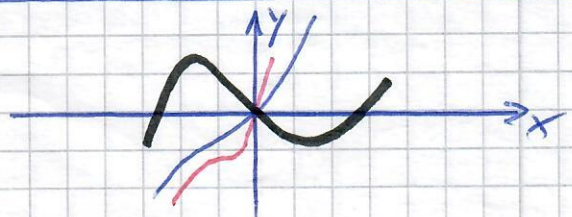
\rightarrow eine wagg. T.

$$c) f(x) = \frac{1}{3}x^3 + x^2 + ax$$

$$f'(x) = x^2 + 2x + a$$

$$\text{u.B. } 0 = x^2 + 2x + a$$

$$\underline{\underline{x_{1,2} = -1 \pm \sqrt{1-a}}}$$



falls $a > 1$

\rightarrow keine wagg. T.

falls $a = 1$ \rightarrow eine w. T.

falls $a < 1$ \rightarrow zwei w. T.

- 10) a) $f(x) = -x^2$
b) $f(x) = x^4$
c) $f(x) = \sin(x)$
d) $f(x) = x^3$
e) $f(x) = -x^2 - 1$