$$f(x) = 2x \cdot \sqrt{9-x^2}$$

1) $DB = \{x,\ x \in R,\ |x| \leqq 3\}$

$P_y(0|0)$

Punktsymmetrie:

$$f(x) = -f(-x)$$

$$2x\sqrt{9-x^2} = -\left(2\cdot(-x)\cdot\sqrt{9-(-x)^2}\right)$$

$$-\shortparallel- \quad = 2x\sqrt{9-x^2} \quad \text{wahr}$$

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2) Nullstellen:

$$0 = 2x \cdot \sqrt{9-x^2}$$

$$\underline{\underline{x_{0_1} = 0}} \qquad \underline{\underline{x_{0_2} = 3;\ x_{0_3} = -3}}$$

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3) Unstetigkeiten: keine

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4) lokale Extrempunkte:

$$f'(x) = 2\cdot\sqrt{9-x^2} + \cancel{2}x \cdot \frac{-2x}{\cancel{2}\sqrt{9-x^2}}$$

$$= \frac{2(9-x^2) - 2x^2}{\sqrt{9-x^2}}$$

$$= \underline{\underline{\frac{18-4x^2}{\sqrt{9-x^2}}}}$$

Sün. B: $0 = 18-4x^2 \curvearrowright \underline{\underline{x_{E_1} = \sqrt{4{,}5};\ x_{E_2} = -\sqrt{4{,}5}}}$

$$f''(x) = \frac{-8x\sqrt{9-x^2} - (18-4x^2)\cdot\frac{-x}{\sqrt{9-x^2}}}{(9-x^2)}$$

$$= \frac{\frac{-8x(9-x^2) + (18-4x^2)x}{\sqrt{9-x^2}}}{9-x^2}$$

$$= \underline{\underline{\frac{4x^3 - 54x}{\sqrt{(9-x^2)^3}}}}$$

①

zu 4:

S.ü.h.B. $f''(\sqrt{4{,}5}) = \dfrac{4\cdot(\sqrt{4{,}5})^3 - 54\cdot\sqrt{4{,}5}}{\sqrt{(9-(\sqrt{4{,}5})^2)^3}}$

$$= \frac{18\cdot\sqrt{4{,}5} - 54\cdot\sqrt{4{,}5}}{\sqrt{(9-4{,}5)^3}}$$

$$= \frac{-36\cdot\sqrt{4{,}5}}{\sqrt{(4{,}5)^3}} = \underline{\underline{-8}} < 0$$

$\Rightarrow$ Max. $\Rightarrow$ $\underline{\underline{P_H(2{,}12\,|\,9)}}$

$f''(\sqrt{4{,}5}) = 8 > 0$ (aufgrund Symm.)

$\Rightarrow$ Min $\Rightarrow$ $\underline{\underline{P_T(-2{,}12\,|\,-9)}}$

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5) Wendepunkte:

$$f''(x) = \frac{4x^3 - 54x}{\sqrt{(9-x^2)^3}}.$$

S.ü.u.B: $0 = 4x^3 - 54x$

$0 = x(4x^2 - 54)$

$\underline{\underline{x_{w_1} = 0}}$ $\quad x_{w_{2,3}} = \pm\sqrt{13{,}5}$

↳ entfallen wg. DB!

S.ü.h.B

Monotonie: $][-2\,|\,0)$: $f''(x) = \dfrac{x(4x^2-54)}{\sqrt{(9-x^2)^3}} = \dfrac{(-)\cdot(-)}{(+)} > 0$

$\Rightarrow$ mon. wachsend

$][0\,|\,2]$: $f''(x) = \dfrac{(+)\cdot(-)}{(+)} < 0 \Rightarrow$

$\Rightarrow$ mon. fallend

$\Rightarrow$ Monotoniewechsel! $\Rightarrow$ f' hat hier Exp.

$\Rightarrow$ f hat hier <u>WP</u>!

6) Verhalten $x \to \pm\infty$:
entfällt!

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7) Graph: