1

$f(x) = 2x \cdot \sqrt{9-x^2}$

1) $DB = \{x, x \in \mathbb{R}, |x| \leqq 3\}$

$P_y(0|0)$

Punktsymmetrie:

$$f(x) = -f(-x)$$

$$2x\sqrt{9-x^2} = -\left(2\cdot(-x)\cdot\sqrt{9-(-x)^2}\right)$$

$$-\|- = 2x\sqrt{9-x^2} \quad \text{wahr}$$

2) Nullstellen:

$$0 = 2x \cdot \sqrt{9-x^2}$$

$x_{0_1} = 0$ $\quad$ $x_{0_2} = 3$ ; $x_{0_3} = -3$

3) Unstetigkeiten: keine

4) lokale Extrempunkte:

$$f'(x) = 2\cdot\sqrt{9-x^2} + \cancel{2}x \cdot \frac{-\cancel{2}x}{\cancel{2}\sqrt{9-x^2}}$$

$$= \frac{2(9-x^2) - 2x^2}{\sqrt{9-x^2}}$$

$$= \frac{18-4x^2}{\sqrt{9-x^2}}$$

Süi. B: $0 = 18-4x^2 \Leftrightarrow x_{E_1} = \sqrt{4{,}5}$ ; $x_{E_2} = -\sqrt{4{,}5}$

$$f''(x) = \frac{-8x\sqrt{9-x^2} - (18-4x^2)\cdot\frac{-x}{\sqrt{9-x^2}}}{(9-x^2)}$$

$$= \frac{\frac{-8x(9-x^2) + (18-4x^2)x}{\sqrt{9-x^2}}}{9-x^2}$$

$$= \frac{4x^3-54x}{\sqrt{(9-x^2)^3}}$$

zu 4:

S. ä. h. B. $f''(\sqrt{4,5}) = \dfrac{4\cdot(\sqrt{4,5})^3 - 54\cdot\sqrt{4,5}}{\sqrt{(9-(\sqrt{4,5})^2)^3}}$

$= \dfrac{18\cdot\sqrt{4,5} - 54\cdot\sqrt{4,5}}{\sqrt{(9-4,5)^3}}$

$= \dfrac{-36\cdot\sqrt{4,5}}{\sqrt{(4,5)^3}} = \underline{\underline{-8 < 0}}$

$\Rightarrow$ Max. $\Rightarrow \underline{\underline{P_H(2,12 \mid 9)}}$

$f''(\sqrt{4,5}) = 8 > 0$ (aufgrund Sym.)

$\Rightarrow$ Min $\Rightarrow \underline{\underline{P_T(-2,12 \mid -9)}}$

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5) Wendepunkte:

$$f''(x) = \frac{4x^3 - 54x}{\sqrt{(9-x^2)^3}}$$

S. ä. n. B: $0 = 4x^3 - 54x$

$0 = x(4x^2 - 54)$

$\underline{\underline{x_{w_1} = 0}}$ $\quad x_{w_{2,3}} = \pm\sqrt{13,5}$

$\hookrightarrow$ entfallen wg. DB!

S. ä. h B

Monotonie: $]-2 \mid 0)$: $f''(x) = \dfrac{x(4x^2-54)}{\sqrt{(9-x^2)^3}} = \dfrac{(-)\cdot(-)}{(+)} > 0$

$\Rightarrow$ mon. wachsend

$]0 \mid 2]$: $f''(x) = \dfrac{(+)\cdot(-)}{(+)} < 0 \Rightarrow$

$\Rightarrow$ mon. fallend

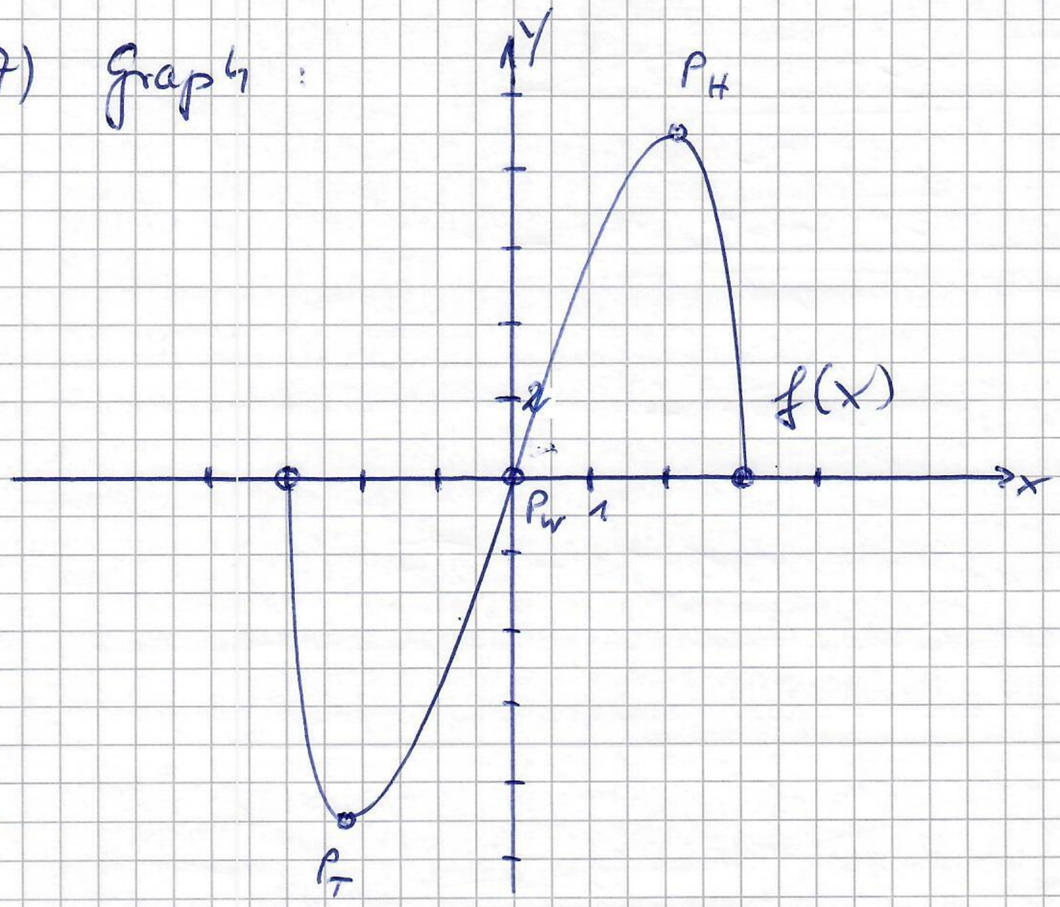
$\Rightarrow$ Monotoniewechsel! $\Rightarrow f'$ hat hier Exp.

$\Rightarrow f$ hat hier WP!

③

6) Verhalten $x \to \pm\infty$:
entfällt!

7) Graph:



$f(x) = (x-2)\sqrt{5-x}$

① $\mathbb{D}B = \{x; x \in R, x \leqq 5\}$; Symm. keine; $P_y(0/-2\sqrt{5})$
$\approx -4{,}5$

② Nst: $0 = (x-2)\sqrt{5-x}$

$\underline{\underline{x_{0_1} = 2 \qquad x_{0_2} = 5}}$

③ Umstel. keine

④ lok. Extrema: $f'(x) = \dfrac{-3x+12}{2\sqrt{5-x}}$

n.B: $0 = -3x+12 \Rightarrow \underline{\underline{x_E = 4}}$

$f''(x) = \dfrac{3x-18}{4\sqrt{(5-x)^3}}$

h.B: $f''(4) = \dfrac{-6}{4} < 0 \Rightarrow \underline{\underline{P_H(4/2)}}$

⑤ WP: $f''(x) = \dfrac{3x-18}{4\cdot\sqrt{(5-x)^3}}$

n.B: $0 = 3x - 18 \Rightarrow x_w = 6 \notin \mathbb{D}B$!

$\Rightarrow$ $\underline{\underline{\text{kein } P_w}}$

⑥ $x \to -\infty$:

$\lim\limits_{x\to-\infty} (x-2)\sqrt{5-x} = \lim\limits_{x\to-\infty}(x-2) \cdot \lim\limits_{x\to-\infty}\sqrt{5-x}$

$= -\infty \cdot +\infty$

$\underline{\underline{= -\infty}}$

⑦