1

KD: $f(x) = x^5 + x^4 - 2x^3 - 2x^2 + x + 1$

1 $D_f = \{x \mid x \in \mathbb{R}\}$, $P_y(0|1)$

Achsen-/Punktsymmetrie:

$$f(x) = -f(-x)$$

$$x^5+x^4-2x^3-2x^2+x+1 = -\left((-x)^5+(-x)^4-2(-x)^3-2(-x)^2-x+1\right)$$

$$= -(-x^5+x^4+2x^3-2x^2-x+1)$$

$$\underline{= x^5-x^4-2x^3+2x^2+x-1}$$

falsch

$$f(x) = f(-x)$$

$$x^5+x^4-2x^3-2x^2+x+1 = (-x)^5+(-x)^4-2(-x)^3-2(-x)^2+(-x)+1$$

$$= \underline{-x^5+x^4+2x^3-2x^2-x+1}$$

falsch

2 Nst: $0 = x^5+x^4-2x^3-2x^2+x+1$

Probieren: $\underline{x_{0_1} = 1}$

$$\Rightarrow (x^5+x^4-2x^3-2x^2+x+1):(x-1) = \underline{x^4+2x^3-2x-1}$$

$$\begin{array}{l} -(x^5-x^4) \\ \hline 2x^4-2x^3 \\ -(2x^4-2x^3) \\ \hline 0-2x^2+x \\ -(-2x^2+2x) \\ \hline -x+1 \\ -(-x+1) \\ \hline 0 \end{array}$$

Probieren: $\underline{\underline{x_{0_2} = 1}}$

$$\Rightarrow (x^4+2x^3-2x-1):(x-1) = \underline{x^3+3x^2+3x+1}$$

$$\begin{array}{l} -(x^4-x^3) \\ \hline 3x^3-2x \\ -(3x^3-3x^2) \\ \hline 3x^2-2x \\ -(3x^2-3x) \\ \hline x-1 \\ -(x-1) \\ \hline 0 \end{array}$$

Probieren: $\underline{\underline{x_{0_3} = -1}}$

$$\Rightarrow (x^3+3x^2+3x+1):(x+1) = x^2+2x+1 = (x+1)^2$$

$$\begin{array}{l} -(x^3+x^2) \\ \hline 2x^2+3x \\ -(2x^2+2x) \\ \hline x+1 \\ -(x+1) \\ \hline 0 \end{array}$$

$\Rightarrow \underline{\underline{x_{0_{4,5}} = -1}}$

$$L_0 = \{\underset{[3x]}{-1}, \underset{[2x]}{1}\}$$

3 Unstetigkeiten: Keine

4 lok. Extrema

$$f'(x) = 5x^4 + 4x^3 - 6x^2 - 4x + 1$$

nB: $0 = 5x^4 + 4x^3 - 6x^2 - 4x + 1$

Probieren: $\underline{\underline{x_{E_1} = 1}}$

$$(5x^4 + 4x^3 - 6x^2 - 4x + 1) : (x - 1) = \underline{5x^3 + 9x^2 + 3x - 1}$$
$$\frac{-(5x^4 - 5x^3)}{9x^3 - 6x^2}$$
$$\frac{-(9x^3 - 9x^2)}{3x^2 - 4x}$$
$$\frac{-(3x^2 - 3x)}{-x + 1}$$
$$\frac{-(-x + 1)}{0}$$

Probieren: $\underline{\underline{x_{E_2} = -1}}$

$$\curvearrowright (5x^3 + 9x^2 + 3x - 1) : (x + 1) = \underline{5x^2 + 4x - 1}$$
$$\frac{-(5x^3 + 5x^2)}{4x^2 + 3x}$$
$$\frac{-(4x^2 + 4x)}{-x - 1}$$
$$\frac{-(-x - 1)}{0}$$

$$\curvearrowright \quad 0 = 5x^2 + 4x - 1 \quad | : 5$$
$$0 = x^2 + \frac{4}{5}x - \frac{1}{5}$$

$$x_{E3,4} = -\frac{2}{5} \pm \sqrt{\frac{4}{25} + \frac{5}{25}}$$
$$= -\frac{2}{5} \pm \sqrt{\frac{9}{25}}$$

$\curvearrowright \underline{\underline{x_{E_3} = \frac{1}{5}}} \; ; \; \underline{\underline{x_{E_4} = -1}}$ $\quad L_E = \{\underset{[2x]}{-1}; \frac{1}{5}; 1\}$

$$f''(x) = 20x^3 + 12x^2 - 12x - 4$$

h.B. $f''(-1) = -20 + 12 + 12 - 4 = 0$ unklar

$f''(1) = 20 + 12 - 12 - 4 = 16 > 0$ Min $\underline{P_T(1|0)}$

$f''(\frac{1}{5}) = 20 \cdot \frac{1}{125} + 12 \cdot \frac{1}{25} - 12 \cdot \frac{1}{5} - 4 \underset{TR}{=} -5{,}76 < 0 \curvearrowright$ Max $\underline{\underline{P_H(\frac{1}{5}|1{,}1)}}$

5 Wendepunkt

$$f''(x) = 20x^3 + 12x^2 - 12x - 4$$

nB $\quad 0 = 20x^3 + 12x^2 - 12x - 4$

Probieren: $\underline{\underline{x_{W_1} = -1}}$ (Vermutung aus 4)

$\curvearrowright (20x^3 + 12x^2 - 12x - 4) : (x+1) = \underline{20x^2 - 8x - 4}$

$-(20x^3 + 20x^2)$

$-8x^2 - 12x$

$-(-8x^2 - 8x)$

$-4x - 4$

$-(-4x - 4)$

0

$\curvearrowright \quad 0 = 20x^2 - 8x - 4 \quad | :20$

$$0 = x^2 - \frac{2}{5}x - \frac{1}{5}$$

$$x_{W_{2,3}} = \frac{1}{5} \pm \sqrt{\frac{1}{25} + \frac{5}{25}}$$

$$x_{W_2} = \frac{1}{5} + \sqrt{\frac{6}{25}} = \frac{1}{5} + \frac{1}{5}\sqrt{6} \underset{TR}{\approx} \underline{\underline{0{,}7}}$$

$$x_{W_3} = \frac{1}{5} - \sqrt{\frac{6}{25}} = \frac{1}{5} - \frac{1}{5}\sqrt{6} \underset{TR}{\approx} \underline{\underline{-0{,}3}}$$

$$f'''(x) = 60x^2 + 24x - 12$$

h.B. $\quad f'''(-1) = 60 - 24 - 12 = 24 > 0 \curvearrowright$ Re-Li-Ku

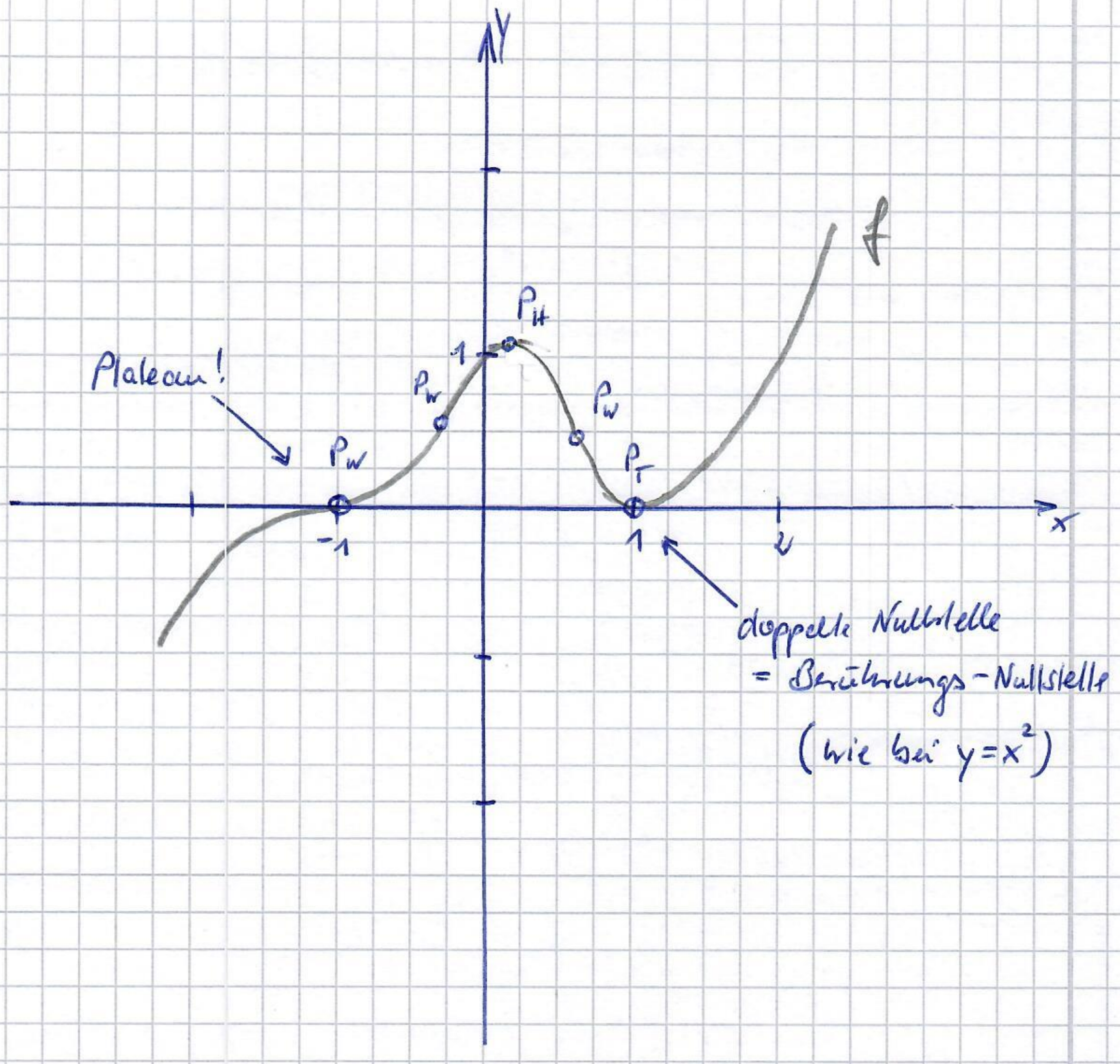
$\underline{\underline{P_{W_1}(-1 \mid 0)}}$

$f'''(0{,}7) = 34{,}2 > 0 \curvearrowright$ Re-Li-Ku $\quad \underline{\underline{P_{W_2}(0{,}7 \mid 0{,}46)}}$

$f'''(-0{,}3) = -13{,}8 < 0 \curvearrowright$ Li-Re-Ku $\quad \underline{\underline{P_{W_3}(-0{,}3 \mid 0{,}6)}}$

6 $x \to \pm\infty$

$$\lim_{x \to \pm\infty} x^5 \cdot \left(1 + \frac{1}{x} - \frac{2}{x^2} - \frac{2}{x^3} + \frac{1}{x^4} + \frac{1}{x^5}\right) = \underline{\underline{\pm\infty}}$$


y
x
f
1
-1
1
2
P_H
P_W
P_W
P_W
P_T
Plateau!
doppelte Nullstelle
= Berührungs-Nullstelle
(wie bei $y=x^2$)